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Mathematics Teachers' Knowledge on Misconceptions and Solution Suggestions: Ratio-Proportion Topic

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The aim of this study is to determine the knowledge of secondary school mathematics teachers about their possible misconceptions about ratio-proportion and their solutions to these misconceptions. In the study, a special case study that is one of the qualitative research methods was conducted. Twelve middle school mathematics teachers were included in the study. Besides, data were collected using a semi-structured interview form developed by the researcher. According to the findings obtained through the content analysis, misconceptions were evaluated in 8 categories and the solutions offered for misconceptions were evaluated in 3 categories. The findings indicate that teachers did not mention some of the misconception types mentioned in the literature on ratio and proportion. In addition, it was observed that approximately half of the participant teachers were aware of the misconceptions mentioned, while the other half expressed only a few types of misconceptions. The solution suggestions of the teachers were generally compatible with the methods stressed in the literature. However, it was determined that these solution suggestions were stated by a small number of teachers. From this point of view, it is thought that it is necessary for teachers to obtain information to increase their awareness of misconceptions that they may observe in students.

Keywords: Ratio-proportion, misconceptions, mathematics teacher

INTRODUCTION

A misconception is defined as a person's understanding of a concept in a way that contradicts its known scientific meaning but makes sense to him/her (Baki, 2015; Yağbasan & Gülçiçek, 2003). It should be known that misconception, which is often confused with the concept of error, is a form of perception which systematically produces errors (Smith, di Sessa & Roschelle, 1993; Zembat, 2015) rather than a momentary mistake. An error can occur because of a misconception, as well as a momentary carelessness, an overlooked situation or a misinterpretation of information (Gates, 2001).

Fisher (1985) stated that misconceptions might arise because of genetic foundations or experiences of the individual and from teaching in school environments. While concepts are being learned, new concepts are built on previous knowledge, and this knowledge sometimes causes difficulties in learning new concepts (Baki & Bell, 1997). When it comes to learning mathematics specifically, the individual's ability to learn concepts in mathematics is related to learning other concepts which are associated with it (Baykul, 2003). Since mathematics has a cumulative structure, an erroneous learning which a student has can cause him/her to make mistakes in the subsequent mathematical subjects or to have misconceptions by causing systematic mistakes (Driver & Easley, 1978; Yılmaz & Yenilmez, 2007). Misconception is an obstacle to conceptual understanding (Minstrell, 1982) and it is very difficult to eliminate misconceptions with traditional methods (Fisher, 1985). The existence of misconceptions can be obvious if students confidently state that the mistakes which they have made are correct and can explain them with their own reasons (Yenilmez & Yaşa, 2008). This form of perception, which finds the misconception logical in itself, can cause students to insist on perpetuating the same misconception (İpekoğlu, 2017). What needs to be done to eliminate misconceptions and to teach mathematics effectively is to teach conceptual and procedural knowledge in a balanced way (Baki, 2015; Birgin & Gürbüz 2009; Soylu & Aydın, 2006). In order to ensure conceptual learning, mathematical concept knowledge should be taught completely, and misconceptions and knowledge deficiencies of students should be identified (Küçük & Demir, 2009). The identified misconceptions should be eliminated in accordance with the nature of the subject.

The Subject of Ratio-Proportion and the Misconceptions Encountered

Ratio is the comparison of multiplicities with the same or different units by dividing them by each other (MoNE, 2018). The comparison of multiplicities belonging to the same measurement space

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is a unitless ratio and the comparison of multiplicities belonging to different measurement spaces is a ratio with units (Şen, 2022). The concept of proportion is defined as "the equivalence of two ratios showing the same relationship" (Lamon, 1995). Ratio-proportion is one of the most important of the many mathematics topics taught at the middle-school level. The conceptual dimension of ratio-proportion provides a bridge to advanced mathematical thinking (Behr, Harel, Post & Lesh, 1992). Difficulties in understanding the topic of ratio and proportion can therefore lead to difficulties in learning advanced mathematics topics.

Kurdal (2016) categorised the errors and misconceptions about ratio and proportion under six headings: misconceptions as conceptual errors, the error of dividing degrees by intervals, lack of percentage logic, failure to understand the relationship between proportion and fraction, failure to establish the relationship between graphs and percentages, and examining the relationship between units one by one. Kaplan, İşleyen and Öztürk (2011) identified the misconceptions encountered at the 6th grade level as misconceptions made while constructing the concept of ratio, students' perception of ratio as real quantity, misconceptions arising from the level of readiness, and the perception of proportional reasoning as direct proportion. Doğan and Çetin (2009) stated that the misconceptions at the 7th and 9th grade levels are not knowing the definition of ratio and proportion, confusing ratios with fraction, number and division, and not being able to determine the types of proportion in given proportionality problems. Karagöz Akar (2009) put misconceptions under three headings: misconceptions about additive and multiplicative associations, misconceptions about covariation and transformation, and students' misconceptions about invariance. Deveci (2021) stated that there are misconceptions reported in the literature such as not understanding the concept of ratio, using the concept of ratio in the same sense as a fraction, perceiving proportional reasoning only as direct proportion, not perceiving the concept of percentage, misconceptions about covariation and transformation, and using additive reasoning instead of multiplicative reasoning in ratio.

Teachers are expected to be aware of the possible misconceptions that students might have in order to deal with them (Chick & Baker, 2005; Shulman, 1987; Szydlik, 2000). However, it has been stated in some studies that teachers do not know what misconceptions can occur and the reasons for them (Çavuş-Erdem, 2016; Gökkurt Özdemır, Bayraktar & Yılmaz, 2017; Kula Ünver, 2016). In the literature, mathematical misconceptions in different subjects have been investigated but there have been few studies which have investigated what kinds of solution teachers have for addressing misconceptions (Bingölbali, 2010; Chick & Baker, 2005; Çavuş Erdem, 2013; İpekoğlu, 2017). Although there is a limited number of studies (Deveci, 2021) which have specifically addressed misconceptions about ratio and proportion, there have been no studies which have addressed misconceptions together with solution suggestions. In the light of this gap, it is thought that the current study will contribute to the literature since it deals with both misconceptions and suggested solutions which are specifically associated with the topic of ratio-proportion.

In this study, secondary-school mathematics teachers' knowledge about misconceptions about ratio-proportion and their suggested solutions for eliminating them are examined. The two research sub-questions were as follows:

1. What are the teachers' views on their students' possible misconceptions about ratio-proportion?
2. What are the solutions that teachers suggest for eliminating misconceptions about ratio-proportion?

METHOD

This study was conducted to examine teachers' current knowledge about students' misconceptions about ratio-proportion and their solutions to these misconceptions using a special case study, one of the qualitative research methods. A special case study is used by a researcher to analyse in depth a program, an event or a process (Creswell, 2013). The case examined here was teachers' knowledge of the misconceptions that students can have about the subject of ratio and proportion and their suggestions for solutions to these misconceptions.

Participants

This study was conducted with the participation of twelve teachers working in different middle schools in a province located in the Central Anatolia Region of Turkey. The study group consisted of teachers who voluntarily participated in the research and were actively engaged in middle-school mathematics teaching. The criterion-based recruitment technique, one of the purposive sampling methods, was used in the selection of the participants. In identifying appropriate participants by this technique, individuals who are generally related to the research subject and have relevant knowledge are included in the study (Yıldırım & Şimşek, 2016). The criterion used in the selection of the participants for the current study was that they were teachers who had taught the subject of ratio-proportion for at least one semester at the secondary-school level. Guest, Bunce and Johnson (2006) stated that over 90% data saturation can be reached with twelve participants in a qualitative study. Twelve teachers were therefore recruited for the study. In order to ensure their anonymity, answer sheets were numbered from 1 to 12 and the notations T1 to T12 were used for presenting the findings. The personal information of the teachers who participated in the study is shown in Table 1.

Table 1. Demographic information about the participants

	Frequency
Gender	
Female	8
Male	4
Years of teaching experience	
1-5 years	4
6-10 years	4
10 years and above	4
Total	12

Data collection tool

An interview form was prepared consisting of open-ended questions to collect data. The questions were in two parts. In the first part, there were questions about the gender and years of teaching experience of the teachers. In the second part, teachers were asked two written questions. The first was 'What are the misconceptions that students can have about the concept of ratio-proportion? Write down all of them and explain them with examples' and the second was 'Write down the suggested solutions you have for the misconceptions which you have stated'. Before the questionnaires were given to the teachers, the opinions of two mathematics educators were sought and the questions were put to the teachers after confirmation had been received of the appropriateness of the questions for the purpose of the study. The question form was applied face-to-face and on a voluntary basis. The average completion time was 40 minutes.

Data Analysis

The data obtained from the participants were analysed using content analysis. In content analysis, the acquired data are coded and categories which are identified from these codes are organized (Corbin & Strauss, 2007; Yıldırım & Şimşek, 2016). The researcher first coded the participants' responses and then organized and categorized the codes according to their similarities and differences. These codes and categories were shared with an independent mathematics educator with experience of conducting qualitative studies. The correspondence between the researchers was examined and the percentage of correspondence was calculated as 90% (Miles & Huberman, 1994), showing that there was therefore consistency in the coding made by the two researchers. As a result of the analysis, the misconceptions and suggested solutions stated by the teachers were tabulated with percentage and frequency information and the findings are presented in the Results section. In addition, explanations and examples given by the teachers for the related misconception are presented.

RESULTS

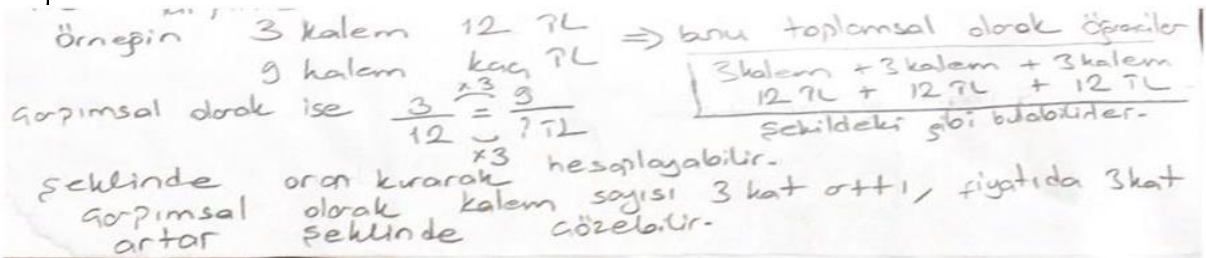
The teachers' views of their students' possible misconceptions about ratio-proportion

The types of misconception stated by the twelve participating secondary-school mathematics teachers about the topic of ratio-proportion are given in Table 2.

Table 2. Misconceptions about Ratio-Proportion given by the teachers

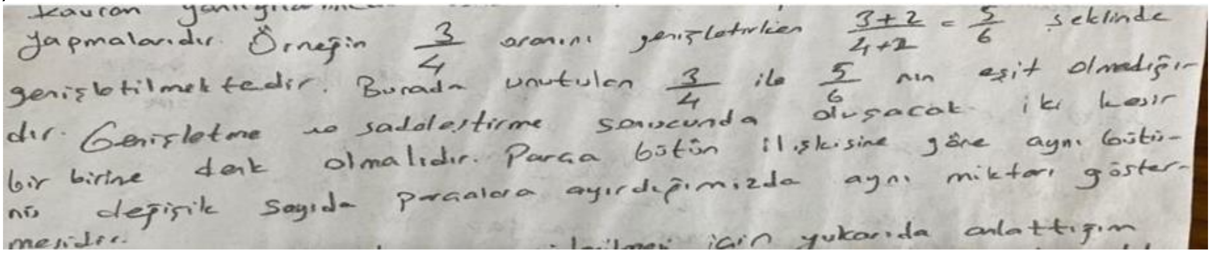
No	*Codes*	*Teachers mentioning the Code*	*f*	*%*
1	Failure to distinguish between additive and multiplicative relationships	T1, T3, T4, T5, T6, T7, T8, T9,T10, T12	10	20
2	Errors in the expansion and simplification of a ratio	T1,T2,T4,T5,T7, T11,T12,	7	14
3	Misconceptions about covariation	T3, T5, T6, T7, T8, T9, T12	7	14
4	Misconceptions about the invariance of a ratio	T1,T3, T5, T6, T7, T11,T12	7	14
5	Misconceptions about perceiving the ratio as the same as the actual amount	T1, T3, T4, T5, T6, T7	6	12
6	Misconceptions about the definition of ratio	T3,T6,T7,T8, T12	5	10
7	Misconceptions about cross-multiplication	T2, 3, T7, T12	4	8
8	Misconceptions related to not knowing the distinction between ratio and fraction	T1, T3,T4,T5	4	8
Total			50	100

Table 2 shows that the misconception most frequently expressed by the teachers among the possible misconceptions in students about ratio-proportion was 'not distinguishing between additive and multiplicative relationships' (f=10). For example, T8 stated that one of the main parts which students did not understand about ratio was the relationships between the elements which make up a ratio. In this context, he/she stated that writing a proportion such as 2/3 = 4/5 was obtained by doing the operation (2+2) / (3+2) = 4/5, that is, the student used additive reasoning instead of multiplicative reasoning. The example in which T3 explained how students should make the correct association was as follows. The teacher stated that in a question such as *'If 3 pencils cost 12 TRY, how much do 9 pencils cost?", the student should reach the result by perceiving the expression 3 pencils + 3 pencils + 3 pencils = 12 TRY + 12 TRY + 12 TRY.* Multiplicatively, he/she stated that since the number of pencils increased 3 times, so the price should also increase 3 times.

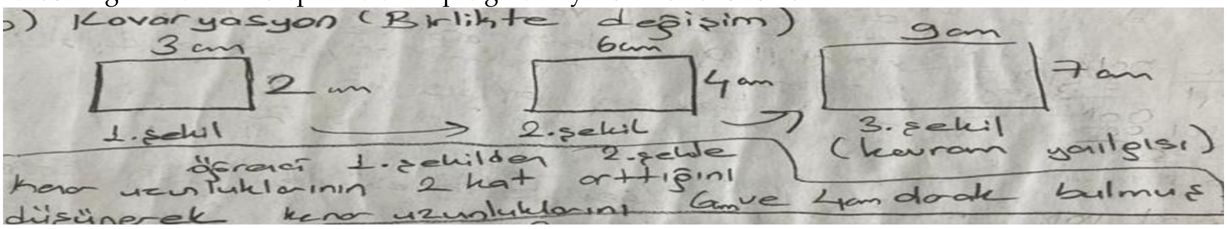

Örneğin 3 kalem 12 TL ⇒ bunu toplamsal olarak öğrenciler
9 kalem kaç TL
3kalem + 3kalem + 3kalem
12 TL + 12 TL + 12 TL
şekildeki gibi bulabilirler.
Çarpımsal olarak ise $\frac{3}{12} \overset{\times 3}{=} \frac{9}{?\,TL}$ ($\times 3$) hesaplayabilir.
şeklinde oran kurarak kalem sayısı 3 kat arttı, fiyatı da 3 kat
çarpımsal olarak
artar şeklinde çözebilir.

The second most common misconception stated by the teachers (f=7) was about the expansion and simplification operations in a ratio. T7 stated that the two fractions which will be formed as a result of expansion and simplification should be equivalent to each other and said that students had problems with the transformations that should be made in the numerator and denominator in these expansion and simplification operations.

T7: ... For example, when expanding the ratio 3/4, it is expanded as (3+2) / (4+2) = 5/6. What is forgotten here is that 3/4 and 5/6 are not equal. The two fractions resulting from expansion and simplification must be *equivalent. According to the part-whole relationship, when we divide the same whole into different numbers of parts, it shows the same amount …*

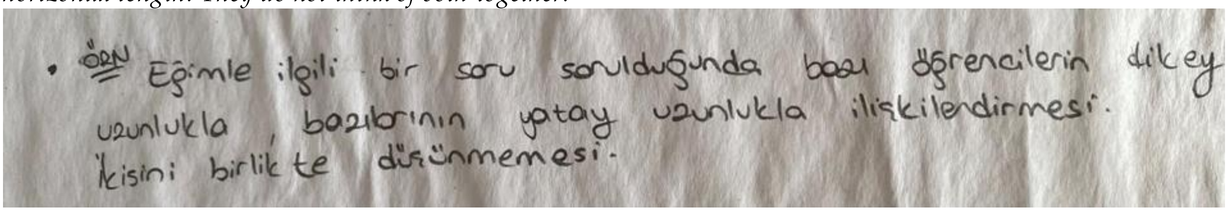

kavram yanılgısı ...
yapmalarıdır. Örneğin $\frac{3}{4}$ oranını genişletirken $\frac{3+2}{4+2} = \frac{5}{6}$ şeklinde
genişletilmektedir. Burada unutulan $\frac{3}{4}$ ile $\frac{5}{6}$ nın eşit olmadığı-
dır. Genişletme ve sadeleştirme sonucunda oluşacak iki kesir
bir birine denk olmalıdır. Parça bütün ilişkisine göre aynı bütü-
nü değişik sayıda parçalara ayırdığımızda aynı miktarı göster-
melidir. ... için yukarıda anlattığım

A third misconception stated by the teachers (f=7) was 'misconceptions about covariation'. Covariation refers to the combined change of two or more than two probabilistic variables and the change to be made in the questions must be of the same type (Deveci, 2021). T6 gave the example of a rectangle with a given short and long side length whis has to be expanded while preserving the shape, and students could show this misconception by replacing the short side and long side with additive reasoning in the third step. The example given by T6 was as follows.



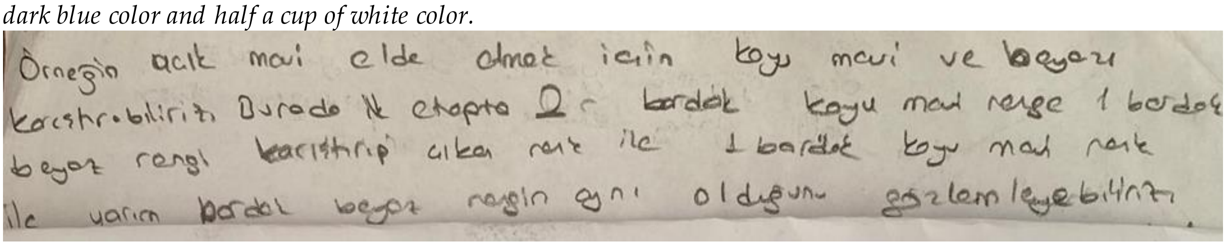
Misconceptions about the invariance of the ratio was another type of misconception stated by the teachers (f=7). T1 gave the example of the representation of the slope of a ramp, and students overlooked the relative value of the vertical axis to the horizontal axis, that is, the ratio, and stated that perceiving the slope only as the measurement of one of the horizontal or vertical axes would be an example of this. He/she stated that a student's inability to perceive that the ratio is a linear relationship between two variables might cause misconceptions:

T1: When a question about slope is asked, some students associate it with vertical length, others with horizontal length. They do not think of both together.

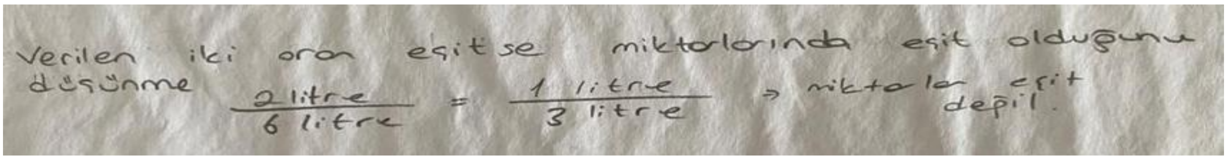

• ÖRN Eğimle ilgili bir soru sorulduğunda bazı öğrencilerin dikey uzunlukla, bazılarının yatay uzunlukla ilişkilendirmesi. İkisini birlikte düşünmemesi.

In addition, regarding the invariance of the ratio, the teachers stated that students had some misconceptions about the ratio showing the invariant relationship between two multiplicities shown as the numerator and denominator. The example given by T5 was as follows.

T5: For example, we can mix dark blue and white to get light blue. Here, in the first place, we can mix two cups of dark blue color with one cup of white color and observe that the resulting color is the same as one cup of dark blue color and half a cup of white color.

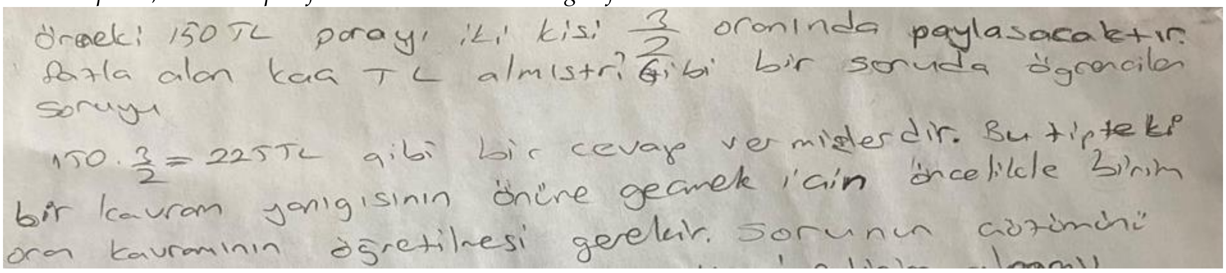

Örneğin açık mavi elde etmek için koyu mavi ve beyazı karıştırabiliriz. Burada ilk etapta 2 bardak koyu mavi renge 1 bardak beyaz rengi karıştırıp çıkan renk ile 1 bardak koyu mavi renk ile yarım bardak beyaz rengin aynı olduğunu gözlemleyebiliriz.

Some of the teachers (f=6) stated that students perceived the concept of ratio not as an expression of comparison but as the real equivalent of the written numbers. T1 gave an example of a misconception such as thinking that if two given ratios are equal, the quantities are also equal.

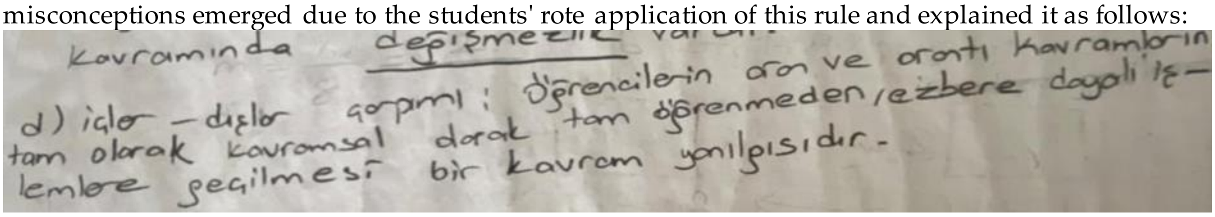

Verilen iki oran eşitse miktarlarında eşit olduğunu düşünme
$\frac{2 \text{ litre}}{6 \text{ litre}} = \frac{1 \text{ litre}}{3 \text{ litre}}$ → miktarlar eşit değil.

Another misconception stated by the teachers (f=5) was about the definition of the concept of ratio and proportion. It was stated that students did not know the definitions of unit and unitless ratio and that they had some misconceptions due to the lack of information in the definition of the concept. T8 gave the following example of this type of misconception that students can have:

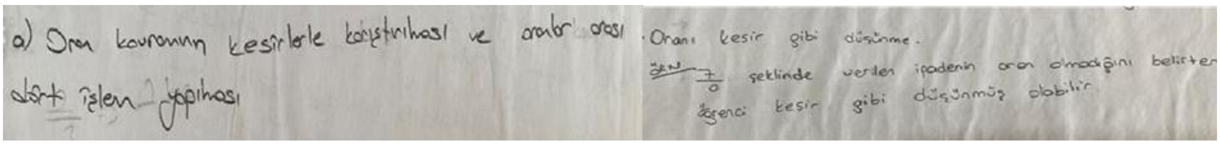
T8: Two people will share 150 TRY in the ratio of 3/2. How many TRY did the person who received more get?' In a question like this, students gave an answer like 150; (3/2) = 225. In order to prevent this type of misconception, the concept of unit rate should be taught first…


Örnek: 150 TL parayı iki kişi $\frac{3}{2}$ oranında paylaşacaktır.
Fazla alan kaç TL almıştır? gibi bir soruda öğrenciler
soruyu
$150 \cdot \frac{3}{2} = 225$ TL gibi bir cevap vermişlerdir. Bu tipteki
bir kavram yanılgısının önüne geçmek için öncelikle birim
oran kavramının öğretilmesi gerekir. Sorunun çözümü

Another misconception (f=4) was about the rule of cross-multiplication. T3 stated that misconceptions emerged due to the students' rote application of this rule and explained it as follows:


kavramında değişmezlik var
d) içler – dışlar çarpımı: Öğrencilerin oran ve orantı kavramlarını
tam olarak kavramsal olarak tam öğrenmeden/ezbere dayalı iş-
lemlere geçilmesi bir kavram yanılgısıdır.

Some teachers (f=4) stated that students had misconceptions because they did not know the difference between ratio and fraction. T4 stated that students confused the concept of fraction and ratio and confused the operations performed in a ratio with those performed in fractions. T1 stated that students perceived the ratio as a fraction and thought that the expression 7/0 was not a ratio.


a) Oran kavramının kesirlerle karıştırılması ve oranlar arası dört işlem yapılması

Oranı kesir gibi düşünme.
$\frac{7}{0}$ şeklinde verilen ifadenin oran olmadığını belirten öğrenci kesir gibi düşünmüş olabilir.

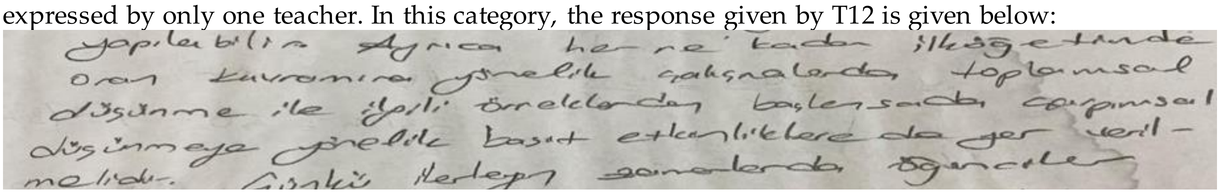
Teachers' solutions for eliminating misconceptions about ratio-proportion

The findings related to the second sub-question of the study which asked for suggested solutions for secondary-school mathematics teachers to address misconceptions about ratio and proportion are given in Table 3.

Table 3. Suggested solutions to misconceptions about ratio and proportion

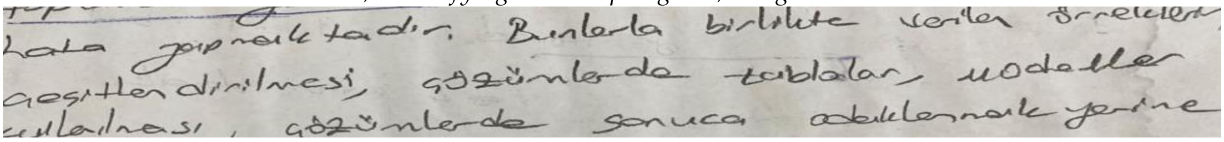
Categories	*No*	*Codes*	*Teachers Indicating the Code*	*f*	*%*
Suggestions for solutions related to pre-teaching	1	Structuring teaching with necessary precautions by knowing possible misconceptions	T3, T7, T8, T12	4	8
	2	Including simple multiplicative relations as well as additive relations in primary education	T12	1	2
Solution suggestions related to the teaching process	1	Concretization of problems with teaching material	T4, T7, T8, T9, T10, T12	6	12
	2	Focusing on how relationships between multiplicities progress in problem solving	T1, T3, T7, T8, T12	5	10
	3	Supporting teaching with real-life problems	T1, T7, T10, T11, T12	5	10
	4	Conceptual teaching (working on definitions of ratio with and without units)	T2,T3,T6,T8,T9	5	10
	5	Including modeling activities	T2,T3,T8,T12	4	8
	6	Replicating different examples with related math topics	T1, T5, T8, T12	4	8
	7	Preparing experiments in the classroom environment	T5,T7,T9,T11	4	8
	8	Explaining confused topics in a comparative way (fractions, percentages, rational numbers and division)	T1, T12	2	4
	9	Teaching through multiple representations	T3, T12	2	4
	10	Falling into cognitive conflict	T4,T12	2	4
	11	Simplifying problems	T6,T10	2	4
	12	Avoiding negative language	T6	1	2
Solution Suggestions Related toPost-Instruction	1	Giving mathematical tasks to students at the end of the lesson to reinforce the subject (homework or a project)	T8,T10,T11	3	6
Total	15			50	100

Table 3 shows that the participating teachers' suggestions for solutions to misconceptions about ratio and proportion were evaluated in three categories: before, during and after the teaching process. In the solution suggestions related to the pre-teaching process, the code 'Structuring the teaching with necessary precautions by knowing the possible misconceptions' constituted 8% of the responses; the code 'Including simple multiplicative relations as well as additive relations in primary education' was expressed by only one teacher. In this category, the response given by T12 is given below:

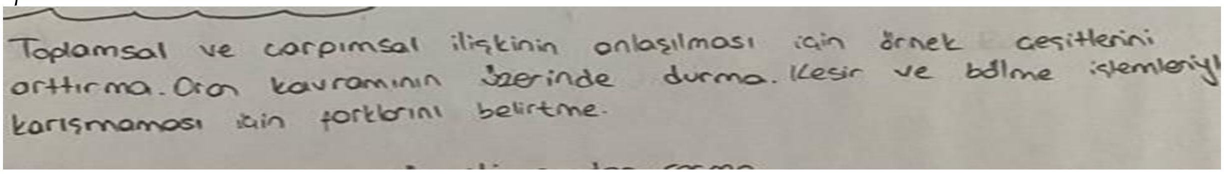

yapılabilir. Ayrıca bence ilköğretimde
oran kavramına yönelik çalışmalarda toplamsal
düşünme ile ilgili örneklerden başlansada çarpımsal
düşünmeye yönelik basit etkinliklere de yer veril-
melidir. Çünkü ilerleyen sınıflarda öğrenciler

Teachers' suggestions for solutions to misconceptions about ratio and proportion related to the teaching process were as follows: 'concretizing the problems with teaching materials' (f=6), 'focusing on how the relations between multiplicities progress in problem solving' (f=5), ' supporting teaching with real-life problems' (f=5), 'conceptual teaching (working on definitions of ratio with and without units)' (f=5), 'including modeling activities' (f=4), 'preparing experiments in the classroom environment' (f=4), 'explaining confused topics in a comparative way (such as fractions, percentages, rational numbers and division') (f=2), 'conducting teaching through multiple representations' (f=2), 'reducing cognitive conflict' (f=2), 'simplifying problems' (f=2) and 'avoiding negative language' (f=1). Some of the responses given by teachers in this category are given below.

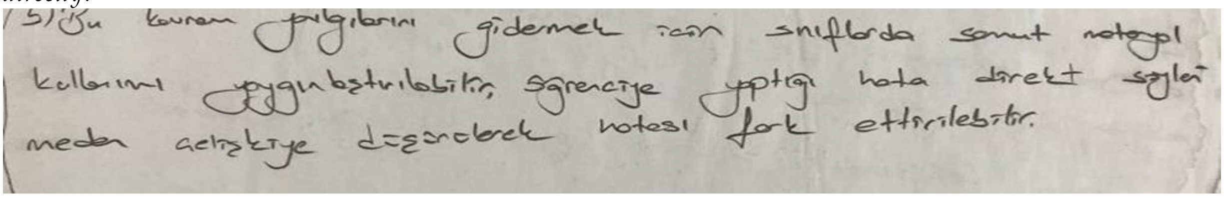
T12: "....In addition to these, diversifying the examples given, using tables and models in solutions..."


hata yapılmaktadır. Bunlarla birlikte verilen örneklerin çeşitlendirilmesi, çözümlerde tablolar, modeller kullanılması, çözümlerde sonuca odaklanmak yerine

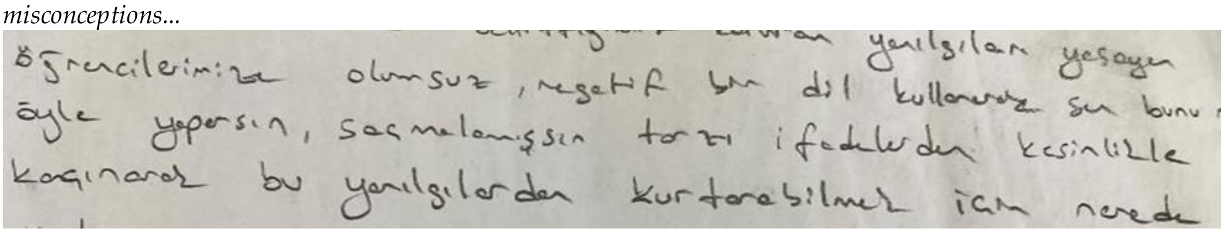
T1: ... Increasing the number of examples to understand the additive and multiplicative relationship. Emphasizing the concept of ratio. Specifying the differences to avoid confusion with fraction and division operations.


Toplamsal ve çarpımsal ilişkinin anlaşılması için örnek çeşitlerini arttırma. Oran kavramının üzerinde durma. Kesir ve bölme işlemleriyl karışmaması için farklarını belirtme.

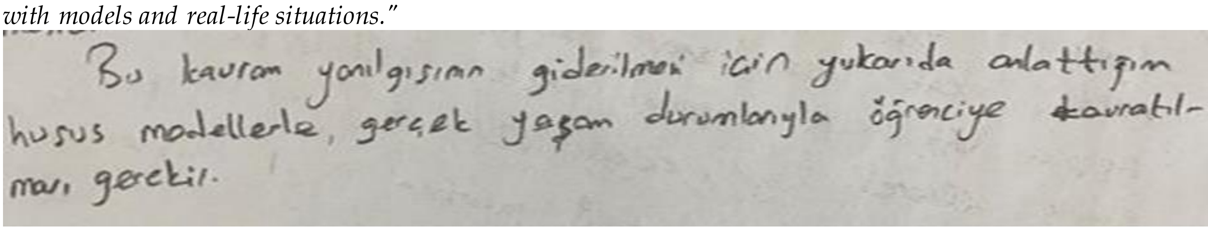
T4: ... In order to eliminate these misconceptions, the use of concrete materials in the classrooms can be made widespread, the student can be made to realize his/her mistake by contradicting him/her without telling him/her directly.


(5) Bu kavram yanılgılarını gidermek için sınıflarda somut materyal kullanımı yaygınlaştırılabilir, öğrenciye yaptığı hata direkt söylenmeden çelişkiye düşürülerek hatası fark ettirilebilir.

T6: ... by not using negative language with our students, avoiding expressions such as 'If you do it like that, you are ridiculous', we can help them understand where they made mistakes in order to save them from these misconceptions...


yanılgıları yaşayan
öğrencilerimize olumsuz, negatif bir dil kullanarak sen bunu öyle yaparsın, saçmalamışsın tarzı ifadelerden kesinlikle kaçınarak bu yanılgılardan kurtarabilmek için nerede

T11: " In order to eliminate this misconception, the issue I mentioned above should be taught to students with models and real-life situations."


Bu kavram yanılgısının giderilmesi için yukarıda anlattığım husus modellerle, gerçek yaşam durumlarıyla öğrenciye kavratılması gerekir.

In the suggested solution give by the teachers related to post-instruction, three of them offered the same solution: 'Giving mathematical tasks to the students at the end of the subject to reinforce the subject'. In this category, the response given by T10 was as follows:

T10: "...homework assignments can be given to students to reinforce the concepts that they confuse."

CONCLUSION and DISCUSSION

In this study, eight different types of misconception were identified in order to determine the knowledge of mathematics teachers about the possible misconceptions which students can have about ratio and proportion and what solutions they could offer to overcome these misconceptions. Students' inability to distinguish between additive and multiplicative relationships was the most common type of misconception expressed by the respondents and the least common type of misconception was not knowing the difference between ratio and fraction. These findings confirm those reported in the literature (Karagöz Akar, 2009; Doğan & Çetin, 2009; Simon & Blume, 1994; Deveci, 2021; Şermetoğlu & Baki, 2019) showing that students have misconceptions about determining additive and multiplicative relationships. Lamon (1995) stated that the concept of ratio inherently involves situations which require multiplicative associations. Students' ability to distinguish between these situations needs to be improved. Misconceptions arise in cases where the connection between the concept of ratio and multiplicative relationship cannot be established (Karagöz Akar, 2009). The teachers stated that students had misconceptions about expansion and simplification. This situation, also called the concept of transformation, means the transformation of numerically different ratios in the representation (Karagöz Akar, 2009). The existence of this situation, which includes the concept of equality, was raised as a misconceptions found in students in this study, which was also reported in similar studies in the literature (Deveci, 2021; Karagöz Akar, 2009). Misconceptions about covariation was another type of misconception stated by the teachers. In a study conducted by Karplus et al. (1983), students were asked to expand a rectangle with dimensions of 2cm and 3cm while preserving its shape and found that the students understood that there was a change in the two dimensions of the rectangle at the same time but did not understand that this change was a multiplicative relationship (Karagöz Akar, 2009).

The participating teachers also stated that students had misconceptions about the definition of ratio. This finding coincides with similar findings in the literature that students could not comprehend the definition of ratio correctly (Deveci, 2021; Umay & Kaf, 2005; Kaplan, İşleyen & Öztürk, 2011). Akkuş Çıkla and Duatepe (2002) conducted a study with pre-service teachers and found that although they could solve questions related to ratio and proportion, they could not define these concepts. Doğruel (2019) found that teachers also had problems and mistakes in the definition of ratio and that a large number of them did not know the definition of ratio. When considered in this context, the fact that teachers have the knowledge that there are misconceptions about the definition of ratio indicates that they are aware of the need for stronger concept teaching. The misconception that the ratio is the same as the actual amount is another misconception expressed by the teachers in the current study. Kaplan et al. (2011) stated that students could not comprehend the ratio as a fraction expression and comparison, and that they took the ratio as an actual quantity in the questions. Similarly, in the current study, it was stated that students had misconceptions about not knowing the distinction between ratio and fraction. Misconceptions about the invariance of the ratio was another misconception stated by the teachers. Invariance is an expression which shows the invariance in the state expressed by a ratio and the invariant relationship between the two multiplicities shown in the numerator and denominator (Simon & Blume, 1994). Another misconception identified in this study was that students have misconceptions about the rule of cross-multiplication. Participating teachers stated that students had misconceptions both when applying the rule and when determining the situations in which the rule should be applied. Akkuş Çıkla and Duatepe (2002) stated that in cross-multiplication, students perform operations by rote rather than as a result of conceptual understanding.

The solutions suggested by the teachers for eliminating the misconceptions of students were evaluated in three categories: pre-instruction, during the instructional process, and post-instruction. First, it was suggested that teachers should be aware of possible misconceptions before the teaching begins and should structure the teaching with the necessary precautions and include simple multiplicative relationships as well as additive relationships in primary education. This would be a very effective method for preventing misconceptions and errors if the teacher is aware of possible student errors related to a subject and plans his/her teaching accordingly (Çavuş Erdem, 2013). The subject of ratio is included in the curriculum for the first time at the sixth-grade level, but students' encounters with additive and multiplicative relationships begin much earlier. Therefore, in order for them to make sense of these relationships, including multiplicative relationships at a simple level in primary education will help students later in their education.

Teachers' suggestions for solutions related to the teaching process were concretizing the problems with teaching materials, focusing on how the relations between multiplicities progress in problem solving, supporting teaching with real-life problems, conceptual teaching (working on definitions of ratio with and without units), including modeling activities, multiplying different examples with related mathematics topics, preparing experiments in the classroom environment, explaining the confused topics in a comparative way (such as fractions, percentages, rational numbers and division), teaching through multiple representations, reducing cognitive conflict, simplifying problems and avoiding negative language. Cornu (1991) stated that misconceptions in students can arise from pedagogical issues such as the content and method of teaching the subject. Şahin and Karakuş (2021) examined methods used by mathematics teachers for teaching ratio and proportion and found that in methods related to the concept of ratio, they used examples expressing what the definition means, and in the method of converting different units to each other, there were examples related to the operational process, and they mostly preferred standard examples. In this respect, it can be said that the findings of this study reported above coincide with the teachers' thinking that they should avoid standard examples and teach with different examples. Suggesting that different disciplines should be utilized in the teaching of ratio, the teachers also stated that the teaching of ratio and proportion would be more explanatory if multiple representations are used. There were some findings that interdisciplinary integration and developmental examples with different representations were limited in studies examining teachers' content knowledge on the subject of ratio and the examples used (Şen, 2022; Şahin & Karakuş, 2021). It can therefore be said that teachers should employ activities which will diversify and increase their students' concept knowledge and application experiences to prevent misconceptions which might occur. Teachers should explain the subjects which are confused with the subject of ratio in a comparative way (f=4) and that the subject should be supported with real-life problems (f=10). Yıldırım Akar (2020) similarly found that students had difficulty in establishing a relationship between one concept and another in the subject of ratio and could not give verbal examples from real life. In this context, it can be said that there was awareness among the teachers of the usefulness of including real-life problems in teaching and using comparative teaching with confused subjects. In general, however, this awareness needs to be more widespread among teachers. In this study, only two of the teachers stated that creating cognitive conflict can be used to eliminate misconceptions, but it has been stated that a good way to eliminate misconceptions could be by confronting students with contradictions and inconsistencies in their own solutions and making them fall into cognitive conflict (İpekoğlu, 2017; Swan, 2001; Şahin, 2011). In this context, it can be said that this highly effective method should be popularized among teachers. The teachers stated that misconceptions can be eliminated by giving students mathematical tasks such as homework and projects at the end of the lesson to reinforce the subject. This finding is consistent with those of other studies in the literature on the resolution of misconceptions (for example, İpekoğlu, 2017).

Some studies in the literature (Doğan & Çetin 2009; Doğruel, 2019) have shown that teachers themselves have deficiencies and misconceptions in their own content knowledge about ratio and proportion. The mathematics teachers participating in this study expressed some of the misconceptions which can be recognised in students. However, it was determined that only approximately half of the teachers were aware of these misconceptions. In other words, half of the teachers participating in the study were able to identify only a few of these misconceptions. It is therefore thought that the awareness of misconceptions about the subject among teachers should be increased. In addition, it was also found that some misconceptions about ratio-proportion, such as not being able to determine the types of proportion in proportion problems, not being able to perceive the concept of percentage, and structural similarity awareness, which are mentioned in the literature, were not mentioned by the participating teachers at all. It should be ensured that all teachers well know these misconceptions by raising awareness about the misconceptions which can be encountered in the subject of ratio and proportion. In addition, according to the results of this study, it can be said that teachers have ideas about ways to solve the misconceptions which they stated, but it was seen that they mostly expressed only a few solution suggestions and that important suggestions were not widespread. In this case, it can be concluded that the teachers had some knowledge about the misconceptions which might be encountered in students, but that they were inadequate in their use of different methods to solve students' existing misconceptions. In this context, in order to increase the effectiveness and quality of teaching, teachers should have knowledge about the misconceptions they will encounter and should know the methods to be used to solve them. It is known that teachers who have a good command of subject matter knowledge can better identify misconceptions which their students might have (Cornu, 1991; Gess-Newsome, 1999; Halim & Meerah, 2002). Teachers therefore need to deepen their subject knowledge. Arican (2019) investigated secondary-school mathematics teacher-candidates' understanding of proportional and disproportionate relationships and their ability to distinguish these relationships from each other. The participants were pre-service teachers who had attended a mathematics education course on fractions, ratio and proportion. The result was that even after the course, some of the trainee teachers had difficulties in representing and interpreting proportional and disproportionate relationships. The researcher stated that all these difficulties seemed to be related to their traditional training in fractions, ratios and proportions. Studies investigating the elimination of misconceptions have shown that teaching with non-traditional methods is quite effective for eliminating misconceptions (Kılıç, Temel & Şenol, 2015; Moss & Case, 1999). For this reason, it is thought that teachers should learn how non-traditional methods can be applied in the course during their university education in order to eliminate misconceptions in students.

It is known that teachers' concept knowledge increases as their working years increase and there are changes in their instructional practices (Sayın, Özdemir & Öner, 2022). In the current study, the responses of teachers with different years of teaching experience were not analysed separately. In future studies, it would be useful to investigate the effect of greater teaching experience on misconception identification and resolution in order to determine the variables affecting this issue and to improve the quality of teaching.

The findings of this study have shown that teachers do not know enough about the misconceptions which students might have about ratio and proportion and the solution proposals they could use to eliminate them. This may be due to the teachers' lack of subject knowledge or the fact that they could not understand the misconceptions of the students because they did not realize the misconceptions they themselves already had. In future studies, the reasons for this situation could be explored and necessary precautions could be recommended.

Declarations

Conflict of Interest

No potential conflicts of interest were disclosed by the author(s) with respect to the research, authorship, or publication of this article.

Ethics Approval

Kırşehir Ahi Evran University Social and Human Sciences Scientific Research and Publication Ethics Committee granted the formal ethics approval.

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Research and Publication Ethics Statement

The study was approved by the research team's university ethics committee of the Kırşehir Ahi Evran University (Approval Number/ID: 2023/04/13. Hereby, we as the authors consciously assure that for the manuscript is fulfilled:

- This material is the authors' own original work, which has not been previously published elsewhere.
- The paper reflects the authors' own research and analysis in a truthful and complete manner.
- The results are appropriately placed in the context of prior and existing research.
- All sources used are properly disclosed.

Contribution Rates of Authors to the Article

The authors provide equal contribution to this work.

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